Non-angelic concurrent game semantics

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Game semantics

Represents programs by their **interaction** with the context:

$$\left[\!\!\left[\underbrace{\begin{array}{l} \lambda x.\,\texttt{if}\ \ x\,\texttt{then}\,\texttt{true} \\ \qquad\quad \texttt{else}\,\bot \end{array}}_{M} \right]\!\!\right] = \underbrace{\left\{ \begin{array}{ccc} \mathbb{B} & \Rightarrow & \mathbb{B} \\ & & \mathrm{q} \\ \mathrm{q} & & \\ \mathsf{tt} & & \\ & & \mathsf{tt} \end{array} \quad , \quad \begin{array}{ccc} \mathbb{B} & \Rightarrow & \mathbb{B} \\ & & \mathrm{q} \\ \mathrm{q} & & \\ \mathsf{ff} & & \\ & & \end{array} \right\}}_{\textbf{strategy}=\text{set of } \textbf{plays}}$$

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Application $(M : A \to B)\,(N : A)$ is represented by **composition**:

interaction ($\circledast$)	(communication on A)
$+$ **hiding**	(of internal communication on A)

$\rightsquigarrow$ Hiding is key to crucial to get $[\![(\lambda x.\,M)N]\!] = [\![M[N/x]]\!]$.

The problem with nondeterminism

Traditional hiding of game semantics only keeps **visible** events.

$$[\![M\,\texttt{choice}]\!] = \texttt{hide}([\![M]\!] \circledast [\![\texttt{choice}]\!])$$

$$= \texttt{hide}\left(\begin{array}{cc} & \mathbb{B} \\ & q \\ q & \\ \mathsf{tt} & \\ & \mathsf{tt} \end{array} \quad , \quad \begin{array}{cc} & \mathbb{B} \\ & q \\ q & \\ \mathsf{ff} & \end{array} \right)$$

$$= \left(\begin{array}{c} \mathbb{B} \\ q \\ \mathsf{tt} \end{array} \quad , \quad \begin{array}{c} \mathbb{B} \\ q \end{array} \right)$$

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$$= \left(\begin{array}{c} \mathbb{B} \\ \color{red}{\mathrm{q}} \\ \color{green}{\mathsf{tt}} \end{array} \quad , \quad \begin{array}{c} \mathbb{B} \\ \color{red}{\mathrm{q}} \\ \phantom{\mathsf{tt}} \end{array} \right) = \left(\begin{array}{c} \mathbb{B} \\ \color{red}{\mathrm{q}} \\ \color{green}{\mathsf{tt}} \end{array} \right) = [\![\mathsf{tt}]\!]$$

$\rightsquigarrow$ Only adequate for **angelic nondeterminism** (may-equivalence).

What we can do

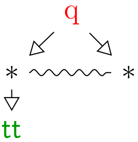
Two existing approaches:

- **Stopping traces** of Harmer and McCusker:
  $\rightsquigarrow$ Tailored for must-equivalence only
- **Playgrounds** of Hirschowitz *et. al* $\rightsquigarrow$ No composition.

In this talk:

- **What?** Obtain non-angelic models in concurrent games.
  $\rightsquigarrow$ Adequacy for weak bisimulation, and a compositional story.

- **How?** Modify hiding to remember hidden divergences.
  $\rightsquigarrow$ They become **internal events**.



Outline of the talk in a table

	Total hiding [RW11]
$[\![N]\!]$	$\mathrm{q} \rightarrowtriangle \mathsf{tt}$
$[\![(\lambda x.\, x) N]\!]$	$\mathrm{q} \rightarrowtriangle \mathsf{tt}$
	$\mathrm{CG}^{\cong}_{\mathsf{total}}$

Outline of the talk in a table

	Total hiding [RW11]	**No hiding**
$[\![N]\!]$	$\begin{array}{c} \mathrm{q} \\ \downarrow \\ \mathrm{tt} \end{array}$	$\begin{array}{ccc} & & \mathrm{q} \\ & & \downarrow \\ & & * \\ & \swarrow & \downarrow \\ * & \sim & * \\ & & \downarrow \\ & & \mathrm{tt} \end{array}$
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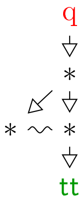
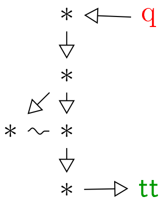
$[\![N]\!]$	$\begin{array}{c} \mathrm{q} \\ \downarrow \\ \mathsf{tt} \end{array}$	
$[\![(\lambda x.\,x)N]\!]$	$\begin{array}{c} \mathrm{q} \\ \downarrow \\ \mathsf{tt} \end{array}$	
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	$\mathrm{CG}^{\cong}_{\mathsf{total}}$	$\mathrm{CG}^{\approx}_{\mathsf{no}}$

Outline of the talk in a table

<table>
<tr><th></th><th>Total hiding [RW11]</th><th>No hiding</th><th>Partial hiding</th></tr>
<tr><td>$[\![N]\!]$</td><td>$$\begin{tikzcd} \mathrm{q} \arrow[d, -{Triangle[open]}] \\ \mathtt{tt} \end{tikzcd}$$</td><td>$$\begin{tikzcd} & \mathrm{q} \arrow[d, -{Triangle[open]}] \\ & * \arrow[dl, -{Triangle[open]}] \arrow[d, -{Triangle[open]}] \\ * \arrow[r, phantom, "\sim"] & * \arrow[d, -{Triangle[open]}] \\ & \mathtt{tt} \end{tikzcd}$$</td><td></td></tr>
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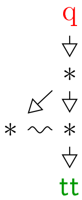
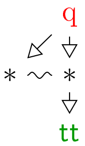
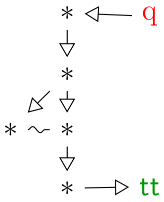
$[\![N]\!]$	$\mathrm{q} \to \mathsf{tt}$		
$[\![(\lambda x.\,x)N]\!]$	$\mathrm{q} \to \mathsf{tt}$		
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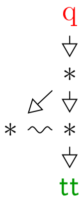
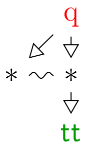
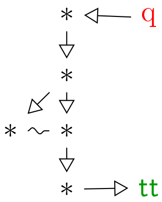
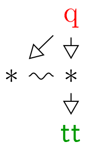
$[\![N]\!]$			
$[\![(\lambda x.\, x)N]\!]$			
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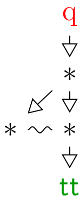
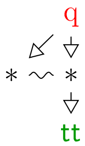
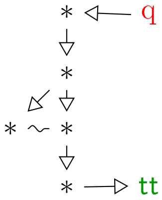
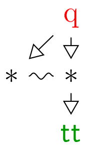
$[\![N]\!]$			
$[\![(\lambda x.\, x) N]\!]$			
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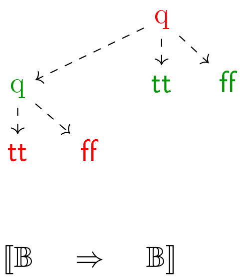
I. $\mathrm{CG}^{\cong}_{\mathsf{total}}$: COVERED STRATEGIES WITH HIDING

Usual concurrent games

Games and covered strategies

Concurrent games are based on **event structures** (es).

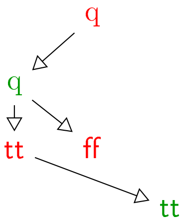
- **Games**: es with polarities A



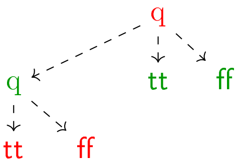
Games and covered strategies

Concurrent games are based on **event structures** (es).

- **Games**: es with polarities A
- **Strategies**: certain event structures S,



$[\![M]\!]$

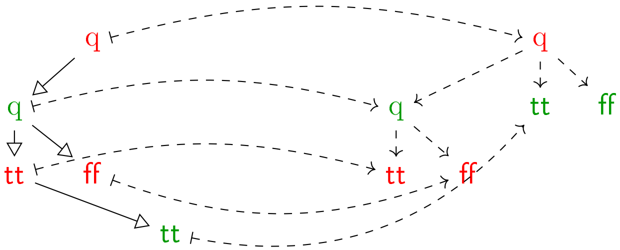


$[\![\mathbb{B} \Rightarrow \mathbb{B}]\!]$

Games and covered strategies

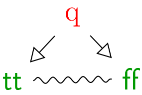
Concurrent games are based on **event structures** (es).

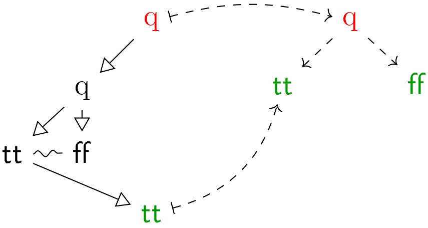
- **Games**: es with polarities A
- **Strategies**: certain labeled event structures $(S, \sigma : S \to A)$



$[\![M]\!]$ $\qquad\qquad$ $[\![\mathbb{B} \Rightarrow \mathbb{B}]\!]$

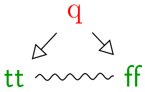
Interaction of strategies

The interaction of $[\![M]\!]$ and $[\![\texttt{choice}]\!] =$  is:

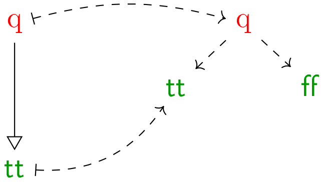


$[\![M]\!] \circledast [\![\texttt{choice}]\!]$ $\qquad$ $[\![\mathbb{B}]\!]$

Interaction of strategies

The composition of $[\![M]\!]$ and $[\![\texttt{choice}]\!]$ = 

is:



$[\![M]\!] \odot [\![\texttt{choice}]\!]$ $\quad\quad$ $[\![\mathbb{B}]\!]$

The category $\mathrm{CG}^{\cong}_{\mathsf{total}}$

Theorem (Rideau-Winskel)

The following is a category $CG^{\cong}_{\mathsf{total}}$:

Objects *Games*

Morphisms *Strategies up to* ***isomorphism***

Composition $\odot$*: Interaction + total hiding.*

$\mathrm{CG}^{\cong}_{\mathsf{total}}$ only supports **angelic** interpretations of nondeterminism.

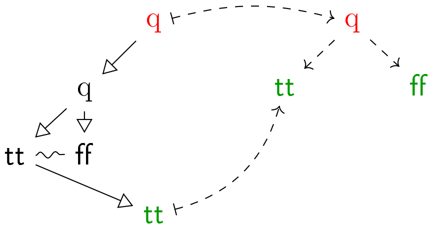
II. $\mathrm{CG}^{\approx}_{\mathsf{no}}$: Uncovered strategies

Remembering every step of the way.

	Total hiding [RW11]	**No hiding**
$[\![N]\!]$	$\begin{tikzcd} \mathrm{q} \arrow[d] \\ \mathrm{tt} \end{tikzcd}$	$\begin{tikzcd} & \mathrm{q} \arrow[d] \\ & * \arrow[d] \arrow[dl] \\ * \arrow[r, phantom, "\sim"] & * \arrow[d] \\ & \mathrm{tt} \end{tikzcd}$
$[\![(\lambda x.\, x) N]\!]$	$\begin{tikzcd} \mathrm{q} \arrow[d] \\ \mathrm{tt} \end{tikzcd}$	$\begin{tikzcd} & * \arrow[d] & \mathrm{q} \arrow[l] \\ & * \arrow[d] \arrow[dl] & \\ * \arrow[r, phantom, "\sim"] & * \arrow[d] & \\ & * \arrow[r] & \mathrm{tt} \end{tikzcd}$
	$\mathrm{CG}^{\cong}_{\mathsf{total}}$	$\mathrm{CG}^{\approx}_{\mathsf{no}}$

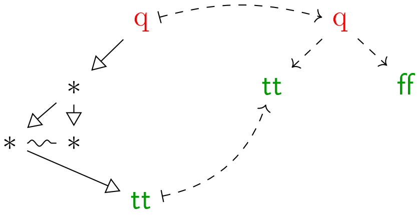
Uncovered strategies

Remember $[\![M]\!] \circledast \texttt{choice}$:



Uncovered strategies

Remember $[\![M]\!] \circledast$ `choice`:



Allow *partial* labelling: $S \rightharpoonup A$ ($*$ are **internal** moves.)

Problem: $\mathfrak{c}_A \circledast \sigma \not\cong \sigma$ if A has a minimal negative move.

Weak bisimulation

Configurations of an uncovered strategy $\sigma : S \rightharpoonup A$ form a LTS:

- $x \xrightarrow{a} y$ if $y = x \cup \{s\}$ and $\sigma s = a$
- $x \xrightarrow{\tau} y$ if $y = x \cup \{s\}$ and σs not defined.

Definition

$\sigma \approx \tau$ when the LTSs $\mathscr{C}(\sigma)$ and $\mathscr{C}(\tau)$ are weakly bisimilar.

Weak bisimulation

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Definition

$\sigma \approx \tau$ when the LTSs $\mathscr{C}(\sigma)$ and $\mathscr{C}(\tau)$ are weakly bisimilar.

For $A = \mathrm{run} \multimap \mathrm{done}$

$$
\begin{tikzcd}
A & \Rightarrow & A \\
\mathrm{run} \arrow[d] & & \mathrm{run} \arrow[ll] \\
\mathrm{done} \arrow[rr] & & \mathrm{done}
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
A & \Rightarrow & A \\
\mathrm{run} \arrow[d] & * \arrow[l] & \mathrm{run} \arrow[l] \\
\mathrm{done} \arrow[r] & * \arrow[r] & \mathrm{done}
\end{tikzcd}
$$

$$\mathbf{c\!\!c}_A \qquad\approx\qquad \mathbf{c\!\!c}_A \circledast \mathbf{c\!\!c}_A$$

Lemma

If A has no mixed polarities conflict, $\mathbf{c\!\!c}_A \circledast \mathbf{c\!\!c}_A \approx \mathbf{c\!\!c}_A$.

A new category

Theorem (C., Clairambault, Hayman, Winskel)

The following is a category $CG_{\mathsf{no}}^{\approx}$:

Objects *Race-free games*

Morphisms *Uncovered secret strategies up to* ***weak bisim.***

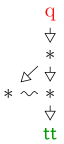
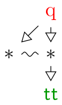
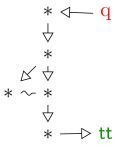
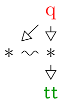
Composition $\circledast$*:* *Interaction with no hiding.*

Problems:

- weak bisimulation is difficult to decide,
- interpretation grows with the term

III. $\mathrm{CG}^{\cong}_{\mathsf{partial}}$: ESSENTIAL EVENTS

Keeping only the essential

	Total hiding [RW11]	No hiding	**Partial hiding**
$[\![N]\!]$			
$[\![(\lambda x.\, x) N]\!]$			
	$\mathrm{CG}^{\cong}_{\mathsf{total}}$	$\mathrm{CG}^{\approx}_{\mathsf{no}}$	$\mathrm{CG}^{\cong}_{\mathsf{partial}}$

To hide or not to hide

A difficult compromise: find a composition $\odot$ such that,

- **hiding** enough so that $\mathfrak{c}_A \odot \mathfrak{c}_A \cong \mathfrak{c}_A$
- **keeping** enough so that $\sigma \odot \tau \approx \sigma \circledast \tau$.

To hide or not to hide

A difficult compromise: find a composition $\circledcirc$ such that,

- **hiding** enough so that $\mathbb{C}_A \circledcirc \mathbb{C}_A \cong \mathbb{C}_A$
- **keeping** enough so that $\sigma \circledcirc \tau \approx \sigma \circledast \tau$.

Definition (Essential event)

An internal event is **essential** when involved in a minimal conflict.

Hiding inessential events of σ results in a strategy $\mathscr{E}(\sigma)$:

$$\mathscr{E}\left(\begin{tikzcd} \mathrm{run} \arrow[d] & \mathrm{run} \arrow[l] & \mathrm{run} \arrow[l] \\ \mathrm{done} \arrow[r] & \mathrm{done} \arrow[r] & \mathrm{done} \end{tikzcd}\right) = \begin{tikzcd} \mathrm{run} \arrow[d] & \mathrm{run} \arrow[l] \\ \mathrm{done} \arrow[r] & \mathrm{done} \end{tikzcd}$$

The category of essential strategies

Lemma

Given $\sigma : S \rightharpoonup A$ an uncovered strategy, we have:

1. $\mathscr{E}(\sigma) \approx \sigma$
2. $\mathscr{E}(\mathbb{c}_A \circledast \sigma) \cong \mathscr{E}(\sigma)$.

As a result, letting $\tau \odot \sigma := \mathscr{E}(\tau \circledast \sigma)$, we get:

Theorem (C., Clairambault, Hayman, Winskel)

The following is a category $CG^{\cong}_{\text{partial}}$:

Objects Race-free games

Morphisms Uncovered secret strat. σ with $\mathscr{E}(\sigma) = \sigma$, up to ***iso***

Composition $\odot$: Interaction with hiding of inessential events

IV. Link with the operational semantics

Interpreting languages in this framework

$$\begin{tikzcd}
 & \text{Source language} \arrow[dl, "{[\![\cdot]\!]_{\mathsf{no}}}"'] & \\
\mathrm{CG}^{\approx}_{\mathsf{no}} & & \mathrm{CG}^{\cong}_{\mathsf{partial}}
\end{tikzcd}$$

$[\![\cdot]\!]_{\mathsf{no}}$: "Operational" model

Interpreting languages in this framework

$$\begin{tikzcd}
& \text{Source language} \arrow[dl, "{[\![\cdot]\!]_{\mathsf{no}}}"'] \arrow[dr, dashed, "{[\![\cdot]\!]_{\mathsf{partial}}}"] & \\
\mathrm{CG}^{\approx}_{\mathsf{no}} \arrow[rr, bend right, "\mathscr{E}(\cdot)"'] & & \mathrm{CG}^{\cong}_{\mathsf{partial}}
\end{tikzcd}$$

$[\![\cdot]\!]_{\mathsf{no}}$: "Operational" model $\quad$ $[\![\cdot]\!]_{\mathsf{partial}}$: "Normal form" model

Automatic adequacy: $[\![M]\!]_{\mathsf{no}} \approx [\![M]\!]_{\mathsf{partial}}$

$\rightsquigarrow$ Picture worked out for nondeterministic PCF & IPA.

Nondeterministic PCF

Given a term $\vdash M : \mathbb{B}$ of ndPCF, form the derivation tree $\mathfrak{t}(M)$.

$$M = (\lambda b.\, \mathtt{if}\, b\, \mathrm{tt}\, \bot)\, \mathtt{choice}:$$

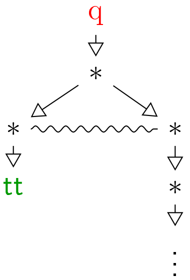
$$\begin{tikzcd}
 & M \arrow[d] & \\
 & \mathtt{if}\,\mathtt{choice}\,\mathrm{tt}\,\bot \arrow[dl] \arrow[dr] & \\
\mathtt{if}\,\mathrm{tt}\,\mathrm{tt}\,\bot \arrow[d] & & \mathtt{if}\,\mathrm{ff}\,\mathrm{tt}\,\bot \arrow[d] \\
\mathrm{tt} & & \bot \arrow[d] \\
 & & \vdots
\end{tikzcd}$$

$$\mathfrak{t}(M)$$

Nondeterministic PCF

Given a term $\vdash M : \mathbb{B}$ of ndPCF, form the derivation tree $\mathfrak{t}(M)$.

$$M = (\lambda b.\, \mathtt{if}\, b\, \mathrm{tt}\, \bot)\, \mathtt{choice}:$$



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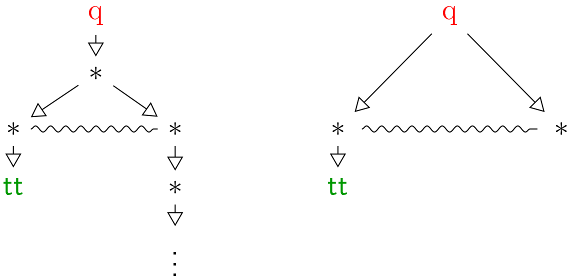

$\mathfrak{t}(M)$ $\qquad$ $[\![M]\!]_{\mathrm{partial}}$

Theorem

For $\vdash M : \mathbb{B}$, $\mathscr{E}(\mathfrak{t}(M)) \cong [\![M]\!]_{\mathrm{partial}}$.

$\rightsquigarrow$ Adequacy for all sorts of convergences (may, must, fair).

Conclusion

Summary.

- Two weakly bisimilar semantics, **related by a map**:
  - one without hiding, ($\simeq$ LTS)
  - one with (partial) hiding ($\simeq$ denotational semantics)

  both **adequate** for **bisimulation**

- "essential events" trick relies on
  - **causal structure**
  - **nondeterministic branching point**
  - **global notion of events**

Future work.

- Full abstraction results for more sophisticated languages?
- Presheaf approach? (eg. model of Tsukada & Ong)

$$\sigma : \mathrm{Plays} \to \mathbf{Set} \qquad \rightsquigarrow \qquad \sigma : \mathrm{Plays} \to \mathbf{PartialOrder}$$